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Thermal Conductivity of Functional Citrus Tree Wood

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Summary. thermal conductivity coefficients have been determined for longitudinal and transverse flow in 4 varieties of fresh Citrus wood using steady-state methods. Equations were developed from which thermal conductivity could be rapidly estimated from moisture content or electrical conductivity. The heat balance of large and small tree trunks on a freezing night has been calculated on the basis of the coefficients.

Thermal Conductivity of Functional Citrus Tree Wood. Heat transfer of the woody stems of trees is chiefly by radiation, convection and conduction. Radiation loss is minimized by the insulating effect of the leafy crown, and convection assumes the dominant role only during the day because the transpiration stream flows most actively then (4); at night thermal conduction, though relatively small, is the principal mode of heat transfer. Although thermal conduction was correctly considered by Raschke (18) to play a minor role in the heat transfer of the plant, under the stress of nocturnal advection freezes and radiation frosts, this is not the case in subtropical plants which bear leaves and fruit during winter. The magnitude of thermal conduction in the wood, relative to its mass, is important in preventing frost damage to citrus trees (24). Therefore, we have determined longitudinal and transverse thermal conductivity coefficients for 4 species of citrus and used these coefficients to analyse the effects of freezing temperatures on 2 cm and 20 cm diameter trees. For this analysis we have used the weather parameters which existed in Weslaco, Texas, from January 9 to 12, 1962 (32).

Thermal conductivity coefficients (K) have been determined on about 70 species of dry woods (10, 13, 14, 20). Across grain K for nearly dry woods, having about 12% moisture, ranges from 0.1 to 0.5 that of freshly cut woods. Wirka (30, 31) was the first to begin studies on the thermal conductivity of green or freshly cut timbers. These studies were subsequently finished by MacLean (12), who later, (13) determined K on 6 species of green wood. The present study deals with the determination of thermal conductivity coefficients (K) and diffusivities (α) for longitudinal and transverse flow and related physical constants of dry, and green or living wood of the 4 principal commercial varieties of Citrus grown in California.

Methods

For estimation of thermal conduction by mathematical analysis Carslaw and Jaeger (5) state that it is necessary to assume the solid is homogeneous and isotropic. Thermal conduction in wood must therefore be experimentally determined because it is (1) heterogeneous, i.e., made up of different tissues of various orientations; is (2) anisotropic, i.e., certain directions are more favorable for the conduction of heat than others.

In the present study, thermal conductivity (K) and diffusivities (α) were determined by the steady-state method of Griffiths and Kaye (10). Large, freshly cut citrus branches of Valencia and Washington Navel orange (Citrus sinensis), Eureka lemon (Citrus limon) and Marsh grapefruit (Citrus paradisi) were collected from trees in the orchard at the Citrus Research Center about 8 AM, and brought to the laboratory in polyethylene bags. Samples one-half cm in length oriented for longitudinal or transverse flow were quickly turned down to fit exactly in the heat transfer apparatus. Heat was supplied from a 6-volt storage battery and a small

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cylindrical electric heater to one end of the wood, the opposite end of which was in contact with a metal cylinder dipping in ice water. Heat loss from the cylinder wall was prevented by a guard-ring of heaters. When steady-state flow was reached, which was in 1 to 2 hours, the temperatures were automatically recorded at the faces of the cylindrical sample with copper-constantan thermocouples and an electronic potentiometer recorder. Density (ρ) was calculated from length and diameter measurements of the fresh woody cylinder made with micrometer calipers. Moisture contents (W) were determined from constant dry weights obtained after drying in a forced draft oven at 65°F and calculated on the fresh weight basis, and by electrical conductivity using a moisture meter. Specific heats (Cv) were calculated as previously shown to be adaptable (28) by the Siebel equation (21) from the moisture contents of the fresh wood.

Thermal conductivity (K) was calculated from the equation

\[ K = \frac{Qd}{(T_0 - T_1)St} \]  

where Q is the heat transferred, d is the thickness of the solid, \( T_0 \) is the temperature of the upper face and \( T_1 \) is the temperature of the lower face, S is the area through which the heat flowed and t is the time. Density (ρ), and percent moisture (W) were also calculated on the dry weight basis for use in equation III. Diffusivity (α) was calculated from the equation

\[ \alpha = \frac{K}{\rho C_v} \]  

**Results**

The thermal conductivity coefficients and diffusivities for longitudinal heat flow parallel to the green-woody axis, and transverse (perpendicular) to the axis are shown in table I. The coefficients for flow parallel to the axis of green wood were on the average about 1.39 times greater, and transverse to the axis 1.59 times greater than the highest K-values recorded for dry wood in the International Critical Tables (14, 22). All values of K were quite similar except those for transverse flow in Eureka lemon. The standard deviation of the mean (σ) shows Eureka lemon differs significantly from the other varieties. The moisture content of lemon wood does not account for the difference in its thermal conductivity. However, its low wet density is significantly different as shown in table II. This suggests a chemical difference, residing possibly in its higher lignin content (26).

The longitudinal thermal conductivity coefficients (K) of green living citrus woods obtained by the Weber method (29) were 1.321 \( \times 10^{-2} \) cal cm\(^{-2}\) sec\(^{-1}\) (C\(^{-1}\) per cm) for grapefruit and 1.199 \( \times 10^{-3} \) for Washington Navel orange wood. They check very well with K values obtained by the Griffiths and Kaye method, i.e., 1.313 \( \times 10^{-3} \) and 1.195 \( \times 10^{-3} \) cal cm\(^{-2}\) sec\(^{-1}\) (C\(^{-1}\) per cm) respectively. The diffusivities (α) determined by the Weber method were 10.69 and 9.21 cm\(^2\)hr\(^{-1}\) respectively. Values of α thus determined confirm the findings by the Griffiths and Kaye method presented in table I, and are in the same range as the sap speeds (2-4 \( \times 10^{-3} \) cm\(^2\)sec\(^{-1}\)) measured by the heat pulse method of Huber.

The thermal conductivity coefficients across-grain of fresh or wet citrus woods range from 0.818 \( \times 10^{-3} \) cal cm\(^{-2}\) sec\(^{-1}\) (C\(^{-1}\) per cm) for lemon to 1.165 \( \times 10^{-3} \) for grapefruit, the highest thermal conductivity of the 4 commercial citrus varieties (table I). Of the highest of the 6 woods (deciduous and evergreen) investigated by MacLean (13), white ash (Fraxinus americana) and white oak (Quercus sp.) were about equal. They averaged 0.892 \( \times 10^{-3} \) cal cm\(^{-2}\) sec\(^{-1}\) (C\(^{-1}\) per cm) which is a little higher than the lowest K value of citrus.

**K-equations.** Thermal conductivity coefficients for heat flow across-grain in citrus, can be empirically calculated from moisture content and specific gravity. These were calculated as those for other woods that have 40% or more moisture by the MacLean equation (13):

\[ K_{40} = \rho (1.39 + 0.038 W) + 0.165 \]  

where \( K_{40} \) = the thermal conductivity in Btu ft\(^{-2}\) hr\(^{-1}\) (F\(^{-1}\) per in)\(^{-1}\), \( W \) = percent moisture based on oven-dry weight, and \( \rho \) = the specific gravity (density) based on oven-dry weight. Calculated values of \( K_{40} \) converted to Cgs units, are given in table I for the wood-moisture contents given in table II. Values of \( K_{40} \) are 2 to 4 times the standard deviation of the mean lower than the experimental values (K).

Heat transfer parallel to the axis empirically calculated as above (\( K_{40} \)) is 4 to 8 times the standard deviation of the mean lower than the experimental values. Correction factors of 1.82, 1.78, 1.71 and 1.65, respectively, are required to obtain the experimental values of K for the varieties in the order shown in table I. Part of this deviation results from the fact that the constants in table II were determined on woods from other trees in a different season the preceding year and because the dimensions for \( C_v \) were made after drying. Caution is clearly indicated for use of MacLean's empirical equation.

The moisture content of the woody structure of the living citrus tree undoubtedly varies with a number

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4 Moisture detector, RC-1B, Delmhorst Instrument Company, Boonton, New Jersey.
5 α is a measure of the temperature change produced in a unit volume by the quantity of heat flowing in a unit time through a unit area of unit thickness with a unit temperature difference between its faces, and is sometimes referred to as thermometric conductivity. The effect of the heater on α is very small (13).

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\( 1 \) Btu ft\(^2\) hr\(^{-1}\) (F\(^{-1}\) per ft) = 0.004134 g-Cal cm\(^{-2}\) sec\(^{-1}\) (C\(^{-1}\) per cm).
of factors. The principal one is perhaps climatic location. In ocean climates, little variation would be expected. But in intermediate, interior and desert districts of southern California, considerable variation between districts would be expected as indicated in the findings for fruit (27). The moisture content varies also with length of time sampling occurs after irrigation. A lemon fruit, for example, in a district having a microclimate intermediate between semi-desert and marine was shown to contain 4 to 5% more moisture 10 days after the tree was irrigated than before (9). The differences in the maximum-minimum moisture contents of a given variety of our wood samples from Riverside where the microclimate is semi-desert, ranged from 2.12 to 5.52%.

MacLean (13) has shown that $K$ increases linearly with moisture percentage and the present determinations confirm this for citrus. MacLean also established a positive correlation between specific gravity and moisture content on the dry weight basis. We have readily confirmed this for citrus, and also found a positive correlation on the fresh weight basis (fig 1). The linear regression equations (wt wt basis) for the curves shown in figure 1 are:

Grapefruit

\[ W = 47.8p - 10.5 \]  

Lemon

\[ W = 33.6p - 0.21 \]  

Valencia Orange

\[ W = 80.6p - 49.9 \]  

Washington Navel Orange

\[ W = 15.4p + 19.5 \]
extension growth of the same barkless wood determined by both electrical and oven-dry methods was sometimes greater, sometimes less than that of mature wood bearing brown bark (table III). But correlation between moisture content determined by the 2 methods was very high \((r = +0.943 \pm 0.028)\). However, the electrical method had a mean systematic error which made \(W \) 2.73% greater than the oven-dry method but with a correction for wood temperature, this reduces to 0.73%. For longitudinal and transverse heat transfer coefficients this is equivalent to 0.044 and 0.030 cal sec\(^{-1}\) cm\(^{-2}\) \(\text{C}^{-1}/\text{cm}\) respectively. Bark contained 20% larger percentages of moisture than wood (table III) but its role in nocturnal thermal conduction will not be considered here. (See Thermal Conductivity of Bark, table III).

The thermal conductivity coefficients for fresh wood \(K_{22}\) can be better calculated by the Andersen equation (1) based on the method of mixtures than by III:

\[
K_{22} = Wk_w + (1 - W) k_k;
\]

where \(k_w\) = thermal conductivity coefficient of water [at 0\(^\circ\), 10, 15, 20 = 1.32, 1.38, 1.40, 1.43 \(\times 10^8\) cal cm\(^{-2}\) sec\(^{-1}\) \(\text{C}^{-1}\) per cm], respectively] and \(K_k\) for oven-dry woods is recorded in table IV. The results of this calculation are also shown in table IV and compare satisfactorily with the measured values of \(K\) on the wet weight basis. Correction coefficients of only 1.18 and 0.92 are required for longitudinal and transverse flow respectively.

### Table III. Comparative Moisture Contents of 4 Varieties of Freshly Cut Citrus Woods Determined by Oven-drying and Electrical-conductivity Methods

<table>
<thead>
<tr>
<th>Variety of wood</th>
<th>Maturity</th>
<th>Wood diameter cm</th>
<th>Moisture content</th>
<th>Electrical-conductivity Wood*</th>
<th>Bark**</th>
<th>Oven-drying Wood***</th>
<th>Δ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Valencia orange</td>
<td>Mature</td>
<td>2.54</td>
<td></td>
<td>35</td>
<td>62.5</td>
<td>33.47</td>
<td>+1.53</td>
</tr>
<tr>
<td>Valencia orange</td>
<td>Green</td>
<td>0.95</td>
<td></td>
<td>36</td>
<td>46</td>
<td>34.62</td>
<td>+1.38</td>
</tr>
<tr>
<td>Wash. Navel orange</td>
<td>Mature</td>
<td>2.54</td>
<td></td>
<td>44</td>
<td>65.1</td>
<td>41.10</td>
<td>+2.90</td>
</tr>
<tr>
<td>Wash. Navel orange</td>
<td>Green</td>
<td>0.95</td>
<td></td>
<td>43</td>
<td>65.5</td>
<td>38.17</td>
<td>+4.83</td>
</tr>
<tr>
<td>Marsh grapefruit</td>
<td>Mature</td>
<td>2.54</td>
<td></td>
<td>47.5</td>
<td>66.7</td>
<td>44.54</td>
<td>+2.96</td>
</tr>
<tr>
<td>Marsh grapefruit</td>
<td>Green</td>
<td>0.95</td>
<td></td>
<td>42</td>
<td>57.5</td>
<td>38.61</td>
<td>+3.39</td>
</tr>
<tr>
<td>Eureka lemon</td>
<td>Mature</td>
<td>2.54</td>
<td></td>
<td>36.5</td>
<td>60</td>
<td>34.95</td>
<td>+1.55</td>
</tr>
<tr>
<td>Eureka lemon</td>
<td>Green</td>
<td>0.94</td>
<td></td>
<td>39</td>
<td>62.5</td>
<td>35.68</td>
<td>+3.32</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td>40.4</td>
<td></td>
<td>60.7</td>
<td>37.64</td>
<td>2.73</td>
<td></td>
</tr>
</tbody>
</table>

* *, **, *** Each value is a mean for 2 limbs collected from the same tree. Bark was removed from both green and mature wood for wood values shown in table.
** Measurement made through the bark.
*** Forced air drying at 65\(^\circ\) to constant weight.
\(Δ\) = difference between moisture content of wood by electrical conductivity and by oven drying.
Heat Budget of Trunks. The thermal behavior of a citrus tree on the calm night of an advective freeze can be reenacted. The root zones lie 0.6 meter and deeper below the soil surface, and it has been shown (32) that even during the 3 days and nights of subfreezing air temperatures, they were in an environment of about 13°. Minimum air temperatures at the height of trees 1.5 meters tall were −8°. Maximum diurnal air temperatures ranged between −2 and 0° for 2.5 days.

During the day a small amount of heat was carried to the trunk and branches by convection in the transpiration stream because air at 0° does not slow or stop the stream (4). At night, however, transpiration in citrus is at a very low value regardless of air temperature. Heat was also conducted from the soil through the roots to the trunk. As a result, trunks wrapped with insulating materials were warmer and very thick rock-wool wraps (15 cm) which had provided trunk surface temperatures 6.5° higher than ambient air temperatures of −6.6° for 5.5 hours (19) kept the trunks of small trees alive for the 3 days of freezing and subfreezing temperatures (11). This success was the result not only of internal convective heating by the transpiration stream (hT), and thermal conduction from the roots (k), but prevention of external free convection (hE), radiation (hR), and evaporative cooling (hE).

External. The radial temperature distribution for an uninsulated grapefruit tree-trunk of 20 cm diameter (φ) and 61 cm length (L) in a medium which was suddenly changed from 0° to −8° is shown in figure 2, and the longitudinal temperature distribution in
Insulated trunks of $\phi = 20$ and 2 cm is shown in figures 3 and 4 respectively. The heat transfer coefficient for free convection ($h_r$), forced convection ($h_f$) at a wind velocity of 91.4 cm sec$^{-1}$, radiation ($h_r$) between different tree parts, and transfer of mass ($h_m$) by different tree parts is shown in table V. For the external trunk in still air $h_r + h_t + h_r = 0.19 \times 10^{-3}$ cal cm$^{-2}$ sec$^{-1}$ C$^{-1}$ and across-grain $K = 0.980 \times 10^{-3}$ cal cm$^{-2}$ sec$^{-1}$ (C$^{-1}$ per cm), but $K$ and $h$ are not directly comparable. The ratio $K/h$, however, has the dimension, cm, which can be thought of as the thickness of a fictitious, external, additional trunk layer of wood having the same thermal resistance as $l/h$, and in this sense the 2 parameters are comparable. For the above values, $K/h = 5.16$ cm which is equivalent to adding a very substantial amount of wood to the trunk radius. A breeze of about 0.2 mi/hr (91.4 cm/sec) causes such a large increase in heat loss through $(h_r)_r$ that $h_r + h_t + h_r = 1.19 \times 10^{-3}$ cal cm$^{-2}$ sec$^{-1}$ C$^{-1}$ and $K/h = 0.824$ cm where $K$ across-grain remains as above (table V). The net effect then, is equivalent to reducing the trunk radius 0.18 cm.

Internal. During a 12 hour night sufficient heat could be supplied by internal forced convection [flow of the transpiration stream (7)] in large trees as shown in table V, to prevent freezing of bare trunks under the given conditions. But at temperatures near 0° which small tree-trunks would rapidly assume, as will be shown later, there is almost no flow (4), freezing and cavitation (23) are likely to occur. Insulation of the trunk answers this problem but poses the question of how rewarming will occur. Table V shows the internal $(h_r)_r$ per tree per day for an average winter day calculated on the basis of the transpiration rates found by Turrell and Austin (24), is greater than needed to offset the trunk losses on this still, severely-cold night. It can also be shown that based on a water usage of 1 acre inch per month during winter by large citrus trees (17), with nocturnal transpiration at 10% of day transpiration (2), the soil at 13° and the trunk at 0°, only an insignificant $2.9 \times 10^3$ cal would be supplied through the xylem and associated tissues in a 12-hour night, provided cavitation did not occur (23), but during a 12-hour day $2.8 \times 10^3$ cal would be supplied by convective heat in the transpiration stream (table V). A cloudy day or a cold day would greatly reduce rewarming of the tree as a whole but should not influence the temperature of the transpiration stream. As shown in table V, the total heat supplied to the trunk by thermal conductivity ($Q_t$) from the soil

![Graph showing temperature distributions after 2, 7, 10 and 17 hours in an insulated (assumed) grapefruit tree trunk.](image-url)

![Graph showing temperature changes over time.](image-url)

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**Fig. 3.** Temperature distributions after 2, 7, 10 and 17 hours in an insulated (assumed) grapefruit tree trunk, $\lambda = 61$ cm, $\phi = 20$ cm, resulting from enthalpic longitudinal heat flow from soil line to first branches and neglecting transverse flow, but assuming a sudden change from maximum air temperature at 0° and trunk temperature (13°) to minimum air temperature at the crotch (−8°) the last night of the 1962 Texas freeze. Elapsed time in hours.

**Fig. 4.** Same as 3 after 2 and 17 hours, except trunk $\phi = 2$ cm.
Table V. Estimated Nocturnal Heat Transfer for Trunk of Grapefruit Trees For Conditions: Minimum Air Temp. -8°, R.H. = 10%, Soil Temp at 31 cm = 13°, Internal Trunk Temp 13°, Trunk Surface Temp 0° to 1°, λ = 61 cm; \( \phi = 2 \) cm, \( a = 1 \) cm; \( \phi = 20 \) cm, \( a = 10 \) cm

<table>
<thead>
<tr>
<th>Item</th>
<th>Cal cm(^{-2}) sec(^{-1}) C(^{-1})</th>
<th>Direction</th>
<th>Time</th>
<th>Wind speed</th>
<th>Trunk size</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_e )</td>
<td>8.13 ( \times 10^{-5} )</td>
<td>Normal to ground</td>
<td>Night</td>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>( h_e )</td>
<td>8.13 ( \times 10^{-5} )</td>
<td>Normal to ground</td>
<td>Night</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>( (h_e)_t )</td>
<td>1.81 ( \times 10^{-4} )</td>
<td>Perpendicular to trunk</td>
<td>Night**</td>
<td>91.4</td>
<td>20</td>
</tr>
<tr>
<td>( (h_e)_t )</td>
<td>5.12 ( \times 10^{-4} )</td>
<td>Perpendicular to trunk</td>
<td>Night</td>
<td>91.4</td>
<td>2</td>
</tr>
<tr>
<td>( h_r )</td>
<td>8.33 ( \times 10^{-5} )</td>
<td>Inside leaves* to Outside leaves</td>
<td>Night</td>
<td>0</td>
<td>( \ldots )</td>
</tr>
<tr>
<td>( h_r )</td>
<td>8.29 ( \times 10^{-5} )</td>
<td>Inside fruit to Inside leaves</td>
<td>Night</td>
<td>0</td>
<td>( \ldots )</td>
</tr>
<tr>
<td>( h_r )</td>
<td>1.01 ( \times 10^{-4} )</td>
<td>Trunk to inside leaves</td>
<td>Night</td>
<td>0</td>
<td>( \ldots )</td>
</tr>
<tr>
<td>( h_s )</td>
<td>1.230 ( \times 10^{-6} )</td>
<td>Leaf to air</td>
<td>Night</td>
<td>0</td>
<td>( \ldots )</td>
</tr>
<tr>
<td>( h_s )</td>
<td>3.02 ( \times 10^{-6} )</td>
<td>Fruit to air</td>
<td>Night</td>
<td>0</td>
<td>( \ldots )</td>
</tr>
<tr>
<td>( h_s )</td>
<td>9.01 ( \times 10^{-6} )</td>
<td>Trunk to air</td>
<td>Night</td>
<td>0</td>
<td>( \ldots )</td>
</tr>
</tbody>
</table>

Cal cm\(^{-2}\) sec\(^{-1}\) C\(^{-1}\) per cm

\( K_{\text{max}} \) 4 Varieties | 0.980 \( \times 10^{-3} \)  Perpendicular to grain | Night | 0          | \( \ldots \) |
\( K_{\text{max}} \) 4 Varieties | 1.417 \( \times 10^{-3} \)  Parallel to grain     | Night | 0          | \( \ldots \) |

Cal per period per tree**

\( (h_e)_t \) (Transpiration stream) \( (13^\circ\text{-soil}) \) | 2.8 \( \times 10^{4} \)  Up stem | Day | 0          | 20         |
\( (h_e)_t \) (Transpiration stream) \( (13^\circ\text{-soil}) \) | 2.9 \( \times 10^{4} \)  Up stem | Night | 0          | 20         |
\( Q_x \) Grapefruit | 7.2 \( \times 10^{4} \) (total) | Up stem | Night | 0          | 20         |
\( Q_x \) Grapefruit | 7.2 \( \times 10^{4} \) (total) | Up stem | Night | 0          | 2          |

* Inside leaves assumed 1° warmer than outside leaves. Inside leaves at air temp.
** 12-hour day and 12-hour night = 2 periods. Velocity of sap in winter = 0.03 to 0.2 cm sec\(^{-1}\) (7).

through the roots below the 30 cm level, is too meagre in small bare trunks \( (\phi = 2 \) cm) to offset the nocturnal heat loss during a 12-hour night, but \( Q_s \) is more than adequate in trunks of large trees \( (\phi = 20 \) cm) even in bare trunks if there is no wind.

Specific Heat. The Siebel equation \( C_p = 0.008W + 0.20 \) first established that the theory of the method of mixtures was practical because of the high specific heat of water. Dunlap determined \( C_p \) on 20 species of oven-dry wood of widely different physical characteristics. The average \( C_p = 0.327 \) cal g\(^{-1}\) C\(^{-1}\). He established that \( C_p = 0.266 + 0.00116v \) where \( v = ^\circ\text{C} \), and 0.266 (cal g\(^{-1}\) C\(^{-1}\)) is measured at 0\(^\circ\text{C} \). Stark found \( r = +0.948 \) between percentage moisture and \( C_p \) of apple wood which yielded a regression equation \( C_p = 0.00566W + 0.3809 \). These 3 works have been used to develop a new equation for the mixture method, \( C_p_{\text{total}} = (C_p \cdot n_{\phi} \cdot X \cdot W) + [(1.000 - W) \cdot (0.266 + 0.00116v)] \) where \( C_p_{\text{total}} \) is the specific heat of wet or freshly cut wood. The results have been calculated at 0\(^\circ\text{C} \) and are recorded in table IV.

Enthalpy, \( K \). If the mass \( (M) \) of the trunk of a large citrus tree is \( 1.99 \times 10^4 \) g, and that of a small tree is \( 2.00 \times 10^2 \) g (24), the specific enthalpy \( (H_x) \) calculated from table II is \( 1.02 \times 10^4 \) cal trunk\(^{-1}\) C\(^{-1}\) in the former and \( 1.02 \times 10^2 \) in the latter. If the trunks were at a maximum of 0\(^\circ\text{C} \) during daylight as on the first day and \(-5.6\)° minimum the following night (32) the total heat available in the trunk of the large tree was \( 5.71 \times 10^4 \) cal compared with \( 5.71 \times 10^2 \) in the small tree which while adequate in the former, is far from adequate to meet the heat demand necessary to prevent freezing in the latter (table V).

Also the surface-mass ratio:

\[ A_s/M \]

decreases with tree size, becoming more favorable for the conservation of stored heat in large trees on winter nights.

\[ A_s = \frac{2A}{a} \]
is transformed to cylindrical coordinates using \((a \cos \theta)\) and \((a \sin \theta)\) respectively, where \(a = \text{radius}\), we obtain:

\[
\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} - \frac{1}{\alpha} \frac{\partial v}{\partial t} = 0
\]

where the diffusivity, \(\alpha\), is defined as in equation (II), \(v\) and \(t\) as in equation (I), and \(x, y, z\) are coordinate lengths. If the plane defined by \(x\) and \(y\)

During a 12-hour night, a tree trunk was assumed protected to the first branches with 61 cm of perfect insulation where microclimate temperature parameters were: air, \(-8^\circ\); trunk center, \(13^\circ\); trunk surface at the crotch, \(0^\circ\); root zone (30.5 cm below soil surface) averaged \(13^\circ\) for a 24 hour period (32).

For the above parameters, the longitudinal, nocturnal steady state flow is 23.1 cal through 1 cm² of grapefruit tree trunk or \(7.2 \times 10^{2}\) and \(7.2 \times 10^{2}\)

Table VI. Estimation of the Time Required for Living Citrus Branches of Various Sizes to Undergo Temperature Change from Day-Maxima \((0^\circ)\) to Night-Minima \((-8^\circ)\) by Radial Heat Transfer Under Freeze Conditions of 1962

<table>
<thead>
<tr>
<th>Radius ((a)) cm</th>
<th>(\alpha) cm²/hr Grapefruit</th>
<th>Time required ((t)) Sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>11.1</td>
<td></td>
</tr>
<tr>
<td>0.50</td>
<td>44.4</td>
<td></td>
</tr>
<tr>
<td>1.00</td>
<td>8.314</td>
<td>177.6</td>
</tr>
<tr>
<td>2.00</td>
<td>710.4</td>
<td></td>
</tr>
<tr>
<td>3.00</td>
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The solution of this heat transfer equation can be approximated without resort to Bessel expansions by use of the Fourier number, \( T = 0.41 \), and

\[
t = \frac{0.41 \, a^2}{\alpha_c} \quad \text{XIV}
\]

which has been solved for various values of \( a \), which are recorded in table VI. These data for the time, \( t \), required for a given temperature change in a branch (cylinder) of radius, \( a \), plot a straight line on log-log paper (fig 6). The equation for this line is

\[
\log t = 2 \log a - 1.244 \quad \text{XV}
\]

and this suggests that a similar transformation will obtain for other parameters.

Thermal conduction is affected by the boundary layer. Both free convection, forced convection, and transpiration conductances are modified by this layer which in turn modifies the surface temperature. Surface temperature of the woody cylinder is also a function of the radiation characteristics and emissivity of the system. Thus the position of the woody axis relative to the soil surface determines its heat transfer properties. Farber and Rennat (8) showed that the conductance (\( h \)) decreased in the first 15 cm of length when the position of an infinite cylinder was varied from \( 0^\circ \) with the horizontal, to \( 90^\circ \). The film coefficient of heat transfer showed a similar distribution, reaching the maximum decrease at about \( 75^\circ \) with the horizontal. In still air, cylinders 30.5 cm long lost heat more rapidly in the horizontal position or close to the horizontal than in the vertical. For longer cylinders, the effect diminished with length.

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They also thank Professor Todd Crawford, Department of Agricultural Engineering, University of California, Davis, and Dr. David Gates, Director, Missouri Botanical Garden for critical discussions of the manuscript.10

Literature Cited


9 A log-plot of the data for spheres (28) is also linear.

10 Citrus Research Center and Agricultural Experiment Station paper No. 1796.


