Boundary Layers of Air Adjacent to Cylinders

ESTIMATION OF EFFECTIVE THICKNESS AND MEASUREMENTS ON PLANT MATERIAL

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ABSTRACT

Using existing heat transfer data, a relatively simple expression was developed for estimating the effective thickness of the boundary layer of air surrounding cylinders. For wind velocities from 10 to 1000 cm/second, the calculated boundary-layer thickness agreed with that determined for water vapor diffusion from a moistened cylindrical surface 2 cm in diameter. It correctly predicted the resistance for water vapor movement across the boundary layers adjacent to the (cylindrical) inflorescence stems of Xanthorrhoea australis R. Br. and Scirpus validus Vahl and the leaves of Allium cepa L. The boundary-layer thickness decreased as the turbulence intensity increased. For a turbulence intensity representative of field conditions (0.5) and for \( v_{\text{wind}} d \) between 200 and 30,000 cm\(^2\)/second (where \( v_{\text{wind}} \) is the mean wind velocity and \( d \) is the cylinder diameter), the effective boundary-layer thickness in centimeters was equal to \( 0.58 \sqrt{d/v_{\text{wind}}} \).

Boundary-layer theory (3, 17, 29) has been applied to the study of the energy and gas fluxes for leaves, which are generally treated as flat plates (5, 8, 21, 24, 28). However, many plant parts such as stems, branches, and certain leaves (e.g., onion) are actually cylindrical. Considerable photosynthesis and presumably water loss as well occurs for the (cylindrical) branches and stems of Cercidium floridum Benth (1), Populus tremuloides Michx. (30), and other plants (1, 22). Moreover, stems and branches often represent 30 to 50% of the above ground surface area of trees and shrubs (1, 34). The present objective was to develop expressions, as simple as possible, describing the thickness of the boundary layers adjacent to cylinders and to compare the resulting predictions of boundary-layer resistance with measurements on water vapor diffusion from cylindrical plant material.

We will consider a cylinder of diameter \( d \) whose axis is normal to the free-stream wind velocity \( (v_{\text{wind}}) \). On approximately the upwind half of the cylinder, there exists a laminar boundary layer which is analogous to the boundary layer adjacent to flat plates and which can be theoretically analyzed (13, 15, 16, 29). On the rear portion of a cylinder, the boundary layer tends to separate from the surface, adverse velocity gradients develop, vortices are shed, and so the air movement there depends in a complicated way on \( v_{\text{wind}} d \) and other parameters (3, 7, 13, 16, 17, 29). Here we will consider that a hypothetical or effective boundary layer of uniform thickness surrounds a cylinder. This highly simplified approach sacrifices a detailed description; e.g., the complicated dependency of heat transfer on angular position around a cylinder (16, 29) is not incorporated. However, relatively simple relations are obtained for estimating the boundary-layer thickness controlling heat and gas fluxes from cylindrical plant surfaces.

THEORY

We will define a thermal boundary layer of effective thickness \( \delta \) such that the temperature goes from that at the surface of the cylinder \((T_s)\) to that of the ambient air \((T_a)\) across this hypothetical distance. Assuming that there are no heat sources within the boundary layer, and for the case of cylindrical symmetry (and no change along the axis of the cylinder, which is assumed to be infinitely long), the heat conducted per unit area and per unit time in a radial direction to or away from the surface can be represented as follows:

\[
J_H = \frac{k}{r} \ln \frac{r + \delta}{r}
\]

where \( J_H \) is the average heat flux at the surface of a cylinder of radius \( r \), and \( k \) is the thermal conductivity coefficient (13, 16). Heat transfer from objects is also described by \( J_H = h(T_s - T_a) \), where \( h \) is the conventional heat transfer coefficient averaged over the cylindrical surface (8, 13, 16). We can insert this expression for \( J_H \) into equation 1, cancel \( T_s - T_a \) from each side, and rearrange to give

\[
\ln \frac{r + \delta}{r} = \frac{2}{hr} = \text{Nusselt number}
\]

where we have incorporated the convenient aerodynamic parameter known as the Nusselt number \((\text{Nusselt number})\).

Heat transfer experiments generally involve determining relationships between the Nusselt number and another dimensionless parameter known as the Reynolds number, \( v_{\text{wind}} d / \nu \), where \( \nu \) is the kinematic viscosity (0.150 cm\(^2\)/sec at 20 C for dry air, ref. 13). Hilpert (10) found that the Nusselt number for cylinders equaled \( \alpha \times \text{Reynolds number} \), where \( \alpha \) times the Reynolds number raised to the power \( m \), where \( \alpha \) and \( m \) depended on the Reynolds number (as the Reynolds number increased from 1 to 250,000, \( \alpha \) went from 0.891 to 0.0239 and \( m \) from 0.330 to 0.805). Upon solving
equation 2 for δ and using Hilpert’s relationship, we obtain

$$\delta = r \left[ \exp \frac{2}{\alpha (\text{Reynolds number})} - 1 \right]$$  

(3)

Although equation 3 is relatively simple, α and m depend on both d and $\nu^{*\text{ind}}$. Hence, various exponential relations and power series were considered in an attempt to provide an approximate but more convenient formula for the effective boundary-layer thickness. The following relationship was found to agree within ±5% with Hilpert’s data for 1 cm$^2$ sec $\leq \nu^{*\text{ind}}d \leq 30,000$ cm$^2$/sec:

$$\delta_{(cm)} = 0.74 \frac{d_{(cm)}}{[\exp 1/\sqrt{\nu^{*\text{ind}}d_{(cm)}} - 1]}$$  

(4a)

where the subscripts indicate the units. The exponent in equation 4a can be expanded in a power series, which converges rapidly for $\nu^{*\text{ind}}d \geq 200$ cm$^2$/sec and leads to

$$\delta_{(cm)} = 0.74 \sqrt{d_{(cm)} \nu^{*\text{ind}} / (\nu^{*\text{ind}}d_{(cm)})}$$  

(4b)

Equation 4 applies at 20°C; to correct within 1% from 0°C to 40°C, δ can be multiplied by $T_{293}$ ($T$ in degrees Kelvin). A more important problem is turbulence intensity (equation 4 applies for a turbulence intensity of 0.009), which we will consider in detail below.

As a working hypothesis, we will assume that the concentration of water vapor drops from $c_{e_{\infty}}$ at the cylindrical surface to the ambient air value ($c_{e_{0}}$) over the same boundary-layer thickness δ as for heat transfer. The flux of water vapor ($J_{w\nu}$) can be described by Fick’s first law, which in the case of cylindrical symmetry (6, 24) is

$$J_{w\nu} = \frac{D_{w\nu} (c_{e_{\infty}} - c_{e_{0}})}{r \ln (r + \delta)} = \frac{\Delta c_{e_{\infty}}^{\nu}}{R_{\nu}^{\nu}}$$  

(5)

where $J_{w\nu}$ is measured at the cylinder surface, $D_{w\nu}$ is the diffusion coefficient of water vapor (0.25 cm$^2$/sec at 20°C, ref. 24), and $R_{\nu}^{\nu}$ is the resistance of the boundary layer to the movement of water vapor.

**MATERIALS AND METHODS**

A cylindrical sleeve of wet filter paper (Whatman No. 2) provided a convenient source of water vapor. The paper, which had an axial length of 100 cm and a 2.00 cm diameter, was centrally placed around a cotton-filled Perspex tube 30 cm long, with a thin slit on one side. The filter paper was inserted into the slit and kept wet by its contact with the moistened, absorbent cotton. The tube was supported at both ends in a conventionally arranged open-circuit wind tunnel with a 4:1 contraction ratio, a downstream axial fan, downstream louvered vanes to control wind speed, and a test section that was 30 cm wide, 35 cm tall, and 100 cm long. The air stream was normal to the axis of the cylinder, end effects were neglected (32), and it was assumed that the boundary layers had the same profile at any cross section along the central third of the tube where the filter paper was mounted.

The plant material used should ideally meet the following criteria: (a) circular cross section, i.e., no ridges or other surface irregularities; (b) constant diameter for 10 cm along the axis; (c) a high rate of water loss; and (d) constancy of stomatal aperture during measurement. Thirty-cm lengths of the inflorescence stems of a water plant (Scirpus validus Vahl) and the native grass-leaf (Xanthorrhoea australis R. Br. subsp. australis) as well as the leaf of onion (Allium cepa L.) were selected. The pith of the inflorescence stems was partially removed and replaced with water to provide essentially 100% relative humidity in the cylinders, while the hollow onion leaves were filled with moistened cotton for the same reason. Before placing the plant material in the wind tunnel, all but the middle 10 cm were wrapped with adhesive tape and the ends were blocked to limit water loss to the central region. The diameter of the central 10 cm varied from 0.84 to 0.87 cm for Scirpus, from 2.00 to 2.09 cm for Allium, and from 4.11 to 4.13 cm for Xanthorrhoea. Measurement of the resistance of the plant material with a diffusive resistance porometer indicated that the stomatal aperture did not change significantly during the period in the wind tunnel. Experiments were also performed using needles of Pinus radiata Don, although the above criteria were not met. A new group of 30 needles averaging 10.7 cm in length were mounted in the wind tunnel for each condition.

The temperature of the filter paper or plant material was continuously monitored using calibrated thermistors and thermocouples placed just under the surface. $T_{w}$ was the average of surface temperatures determined at 60° intervals around the cylinder at 1 cm in from the end of a test section and at 2-cm intervals along the cylinder at 120° from the frontal stagnation point (all values generally were within ±0.3°C of $T_{0}$). In practice $T_{w}$ was usually obtained 1 cm in and 120° from the front. The water vapor content of the air was measured with a Cambridge Systems EG and G Model 880 dew point hygrometer. Weights of the objects before and after a 10 to 30 min period in the wind tunnel were determined with a Mettler AX 101 thermal balance. The inflorescences of the objects before and after a 10 to 30 min period in the wind tunnel were determined with a Mettler AX 101 thermal balance. The resistance of the plant material with a cylindrical sleeve and a rectangular aperture (0.50 cm wide by 1.00 cm along the axis) was placed over the humidity sensor to ensure that only water vapor which emanated from the cylindrical surface was detected. Calibration was done in the manner of Kanemasu et al. (12), but using perforated cylindrical surfaces of approximately the same radius of curvature as the plant material. For a calibration surface with a diameter of 2.00 cm and a calculated resistance of 50 sec cm, the elapsed time for a porometer measurement (14) was only 5% lower than that for a planar calibration plate of the same resistance.

The wind velocity was routinely measured with the test object in place using a hot-wire anemometer. The “probe” was the coiled tungsten filament removed from a flashulight bulb (12, 0.1 amp, 19 ohms when covered and approximately 14 ohms when placed in a wind of 300 cm sec) and mounted on a thin rod in the air stream. The probe was incorporated into a conventional bridge circuit, and the instrument was calibrated in the wind tunnel using a Pitot-static tube and a Fuss inclined microanometer to determine the wind velocity. A DISA 55A25 hot-wire probe, a 55D05 constant temperature anemometer circuit, and a 55D15 linearizer were also used to measure $\nu^{*\text{ind}}$, which was constant within ±1%. for the place of measurement.

Turbulence intensity is defined as $\sqrt{r^{2}, \nu^{*\text{ind}}}$, where $r^{2}$ is the instantaneous fluctuation from the mean wind velocity, $\nu^{*\text{ind}}$, (2, 15, 19, 29, 31). Thus $\sqrt{r^{2}, \nu^{*\text{ind}}}$ represents the standard deviation of the wind speed from its mean (2). To create a turbulence intensity of 0.01 at the position of the test object, a grid of 0.3-cm diameter wires spaced 1.5 cm apart was placed 70 cm upwind of it. Higher turbulence intensities were created by using rectangular parallelpipeds (5 cm wide, 25 cm tall, and 12 cm along the wind direction).

**RESULTS**

When the wind velocity was increased 100-fold, the rate of weight loss from a moistened cylindrical surface increased 10-

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Table I. Calculation of the Resistance for Water Vapor Loss and Effective Boundary-Layer Thickness Adjacent to a Cylindrical Piece of Wet Filter Paper

The cylinder diameter was 2.00 cm and the axial length of the wet filter paper was 10.0 cm, giving a surface area for water evaporation of 62.8 cm². The turbulence intensity was 0.01, \( T_w \) was 20.1 ± 0.3, \( D_{aw} \) was set equal to 0.25 cm² sec⁻¹, and \( r \) was 1.00 cm. The boundary-layer resistance equals \( \Delta c_{aw}^{r} J_{aw} \), which is \( (r D_{aw}) \ln \left( r + \delta' / r \right) \) by equation 5.

<table>
<thead>
<tr>
<th>( \omega_{wind} ) (cm/sec)</th>
<th>( \rho_{aw} ) (g/cm³)</th>
<th>( T_s ) (°C)</th>
<th>( c_{aw}^{r} ) (µg/cm² cm/sec)</th>
<th>( \Delta c_{aw}^{r} ) (µg/cm² cm/sec)</th>
<th>Weight Loss Over 20 min</th>
<th>( J_{aw} ) (g/cm² cm/sec cm)</th>
<th>( \rho_{aw}^{r} ) (g/cm² cm)</th>
<th>( \delta ) (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>19.1</td>
<td>16.59</td>
<td>7.62</td>
<td>4.2880</td>
<td>56.9</td>
<td>0.134</td>
<td>0.0342</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>20.1</td>
<td>17.40</td>
<td>7.62</td>
<td>4.2880</td>
<td>56.9</td>
<td>0.134</td>
<td>0.0342</td>
<td></td>
</tr>
<tr>
<td>1000</td>
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<td>17.40</td>
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<td>4.2880</td>
<td>56.9</td>
<td>0.134</td>
<td>0.0342</td>
<td></td>
</tr>
</tbody>
</table>

**Fig. 1.** Comparison of the effective boundary-layer thickness determined from heat transfer data as represented by equation 4a (——) and that calculated from water vapor loss from a wet cylindrical surface 2 cm in diameter (○). The turbulence intensity in the wind tunnel was 0.01.

Table II summarizes data obtained at various wind velocities for cylindrical plant parts. When \( \omega_{wind} \) was increased from 20 to 200 cm/sec, \( R_{aw}^{r} \) decreased in each case. To see whether the decrease in total resistance could be accounted for by a decrease in boundary layer resistance, \( R_{aw}^{bl} \) was calculated using the boundary-layer thickness determined from equation 4a and the relation implicit in equation 5, viz., \( R_{aw}^{bl} = (r D_{aw}) \ln \left( r + \delta' / r \right) \). \( D_{aw} \) was set equal to 0.25 cm² sec⁻¹, the value appropriate for 20 C (the temperature of the plant tissue did not vary appreciably, e.g., \( T_p \) for onion was 19.4 C at 20 cm/sec and 19.6 C at 200 cm/sec, while \( T_w \) was 20.9 ± 0.3 C). The numbers in the last two column of Table II were obtained by subtracting the calculated value of \( R_{aw}^{bl} \) from the measured value for \( R_{aw}^{total} \).

![Diagram](image-url)
that as the turbulence intensity was increased from about 0.005 to 0.7, the effective boundary-layer thickness adjacent to a 2-
cm diameter cylinder went from 0.169 cm down to 0.116 cm for a $v^{\text{wind}}$ of 50 cm/sec and a similar fractional decrease occurred
for a wind velocity of 200 cm/sec.

The boundary-layer thickness calculated from equation 4a
(represented by $\delta_{0.009}$, since the turbulence intensity for Hilpert’s
data averaged 0.009, refs. 15, 29) was compared with that obtained at a turbulence intensity of 0.5 ($\delta_{0.5}$), a turbulence intensity representative for many plants under natural conditions (2). The following values of $\delta_{0.5}$: $\delta_{0.009}$ were obtained for a wet cylindrical surface 2 cm in diameter: 0.81 at a wind velocity of 20 cm/sec, 0.78 at 50 cm/sec, 0.80 at 100 cm/sec, 0.75 at 200 cm/sec, and 0.74 at 400 cm/sec. Thus $\delta_{0.5}$: $\delta_{0.009}$ averaged 0.78, which suggests that the factor 0.74 in equation 4 is too high for field applications, e.g., 0.78 X 0.74 or 0.58 would be more appropriate.

**DISCUSSION**

The effective thickness of the boundary layer of air surrounding cylindrical plant material can be estimated from the ambient wind velocity and the cylinder diameter, paying due regard to the effect of turbulence intensity. Figure 2 shows that the effective boundary-layer thickness decreased substantially when the turbulence intensity was increased up to about 0.1, in agreement with previous findings for heat (4, 15, 33) and mass (4, 19) transfer from cylindrical surfaces. As the turbulence intensity was further increased up to 0.7, the effective boundary-layer thickness decreased more gradually, i.e., $\delta$ became less sensitive to the actual magnitude of the turbulence intensity in the range of values occurring in the field. Plants under natural conditions often experience a turbulence intensity near 0.5 (2), a value which reduced the boundary-layer thickness an average of 22% from the value predicted by equation 4a. Although the effect of a turbulence intensity of 0.5 appeared to increase somewhat as the wind velocity was raised from 20 to 400 cm/sec (for a 2-cm diameter cylinder), for convenience we will simply reduce the factor 0.74 in equation 4 by 22%.

Table III. Effective Boundary Layer Thickness for Various Wind Velocities and Cylinder Diameters

Values in cm were determined with equation 3 using Hilpert’s data (upper numbers) compared with thickness calculated from equation 4a (lower numbers).

<table>
<thead>
<tr>
<th>$v^{\text{wind}}$ (cm/sec)</th>
<th>$d$ (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>13</td>
</tr>
<tr>
<td>50</td>
<td>23</td>
</tr>
<tr>
<td>1000</td>
<td></td>
</tr>
<tr>
<td>320</td>
<td>0.13</td>
</tr>
<tr>
<td>100</td>
<td>0.47</td>
</tr>
<tr>
<td>1000</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>0.72</td>
</tr>
<tr>
<td>10.2</td>
<td>1.3</td>
</tr>
<tr>
<td>32</td>
<td>1.3</td>
</tr>
<tr>
<td>10</td>
<td>1.3</td>
</tr>
<tr>
<td>3.2</td>
<td>1.5</td>
</tr>
<tr>
<td>0.3</td>
<td>1.5</td>
</tr>
<tr>
<td>0.5</td>
<td>0.87</td>
</tr>
<tr>
<td>0.032</td>
<td>0.87</td>
</tr>
</tbody>
</table>

The present approach can describe heat (equation 1), water vapor (equation 5), carbon dioxide, and oxygen fluxes across the air boundary layer adjacent to cylinders. We can readily incorporate specific resistances encountered within a plant. If $\Delta f$ is the concentration drop of species $f$ across some component and $J_f$ is the flux at the cylinder surface, then the resistance of that component, expressed per unit area of the surface, is $\Delta f / J_f$. The tissue resistances of *Xanthorrhoea*, *Allium*, and *Scirpus* for water vapor movement quoted in Table II are actually fairly low compared with other cylindrical plant parts that were checked with the porometer. For *Xanthorrhoea*, the base of the inflorescence stem where the cuticle was not fully developed was used, and so cuticular transpiration was appreciable. Stomates occupy a large fraction of the surface area of onion (20), which helps therefore recommended for estimating the effective boundary-layer thickness surrounding cylindrical plant parts under natural conditions:

\[
\delta_{(em)} = 0.58 \frac{d_{(em)}}{[\exp \left(1/\sqrt{\frac{\nu^{\text{wind}}}{\nu_{(em/sec)}}}\right) - 1]},
\]

\[
\nu^{\text{wind}}d \leq 30,000 \text{ cm}^2/\text{sec}
\]

\[
\delta_{(em)} = 0.58 \sqrt{\frac{d_{(em)}}{\nu^{\text{wind}}/\nu_{(em/sec)}},}
\]

\[
200 \text{ cm}^2/\text{sec} \leq \nu^{\text{wind}}/\nu_{(em/sec)} \leq 30,000 \text{ cm}^2/\text{sec}
\]

The form of the relation for the effective boundary-layer thickness adjacent to a cylinder (equation 6b) is quite similar to an approximate one for flat leaves under field conditions:

\[
\delta_{(em)} = 0.4\sqrt{\frac{d_{(em)}}{\nu^{\text{wind}}/\nu_{(em/sec)}},}
\]

where $l_{(em)}$ is the distance in cm across the leaf in the direction of the wind (5, 18, 21, 24, 26, 27). The same general conclusions hold in the two cases, e.g., the boundary layer will be thinner for smaller objects or at higher wind velocities. Also, the resistance of the boundary layer of air is usually less than that of the plant part which it surrounds. We note further that the surface area per unit volume of a cylinder is inversely proportional to the radius, and so thin cylinders such as pine needles are better suited for photosynthesis on a tissue volume basis than are large cylinders, and also they will have a lower boundary-layer resistance. Large cylindrical plant parts generally serve important roles besides photosynthesis, such as support, conduction, or storage.
account for its low resistance. Scirpus grows in water and so it can usually withstand a substantial rate of water loss.

Many simplifying assumptions have been incorporated into the above analysis. For instance, heat and gas fluxes by natural (free) convection have been ignored. Natural convection is more important at low wind speeds, large diameters, and large temperature differences between the plant tissue and the ambient air (9, 16, 23, 25). For a temperature difference of 5°C (moderately large) and a wind velocity of 10 cm/sec (low), forced convection exceeds natural convection for diameters up to 100 cm for horizontal cylinders (16, 25). For vertical cylinders up to 300 cm tall and a ΔT of 5°C, forced convection dominates free convection for wind velocities greater than 20 cm/sec (16, 23, 25). When the ambient wind velocity is not normal to the cylinder axis, the effective boundary-layer thickness depends on cylinder length (29); as an approximate guide we can let \( v_{\text{wind}} \) be the velocity component normal to the cylinder axis. For moderately tapering plant parts and for noncylindrical cross sections (but still smooth surfaces, ref. 11) we can let \( d \) be the mean diameter as a useful approximation. The above equations are inaccurate within a few diameters distance from the end of a cylinder (32). These conditions must be kept in mind when using equation 6 to estimate effective boundary-layer thicknesses adjacent to cylindrical plant material.

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APPENDIX

Table III compares the effective thickness of the boundary layer of air adjacent to cylinders calculated from Hilpert’s data (10), as represented by equation 3 (upper numbers), with \( \delta \) determined from the approximate but simpler-to-use formulation developed in this paper, equation 4a. As can be seen, the two expressions for \( \delta \) agree to within about \( \pm 5\% \) for 1 cm/sec \( \leq v_{\text{wind}} \leq 32,000 \) cm sec\(^{-1}\). The calculations are for a turbulence intensity of 0.01; to adjust to a turbulence intensity more appropriate for field conditions (0.5), the numbers should be reduced by 22\%. Table III indicates that the effective boundary layer could vary from approximately 0.005 cm up to 4 cm in thickness for the wide range of wind velocities and cylinder diameters occurring for plants.

LITERATURE CITED