Supporting information: calculation of $J_t$ and $P_{bs}$

The C₄ photosynthesis under light limited conditions can be described as a function of the energy requirements for both the C₄ and C₃ cycles:

$$A = \frac{-b+\sqrt{b^2-4ac}}{2a} \quad \text{(Eqn S1)}$$

(Eqn 4.41, von Caemmerer, 2000)

where

$$a = 1 - \frac{7\gamma^*\alpha}{3 \times 0.047} \quad \text{(Eqn S2)}$$

(Eqn 4.42, von Caemmerer, 2000)

$$b = -\left\{ \left( \frac{xJ_t}{2} - R_m + g_{bs}P_m \right) + \left( \frac{(1-x)J_t}{3} - R_d \right) + g_{bs} \left( \frac{7\gamma^*O_m}{3} \right) + \frac{\alpha\gamma^*}{0.047} \left( \frac{(1-x)J_t}{3} + \frac{7R_d}{3} \right) \right\} \quad \text{(Eqn S3)}$$

(Eqn 4.43, von Caemmerer, 2000)

$$c = \left[ \left( \frac{xJ_t}{2} - R_m + g_{bs}P_m \right) \left( \frac{(1-x)J_t}{3} - R_d \right) - g_{bs} \gamma^*O_m \left( \frac{(1-x)J_t}{3} + \frac{7R_d}{3} \right) \right] \quad \text{(Eqn S4)}$$

(Eqn 4.44, von Caemmerer, 2000)

where $J_t$ is total electron transport rate, $R_d$ is non-photorespiratory CO₂ release in light, $R_m$ is the mesophyll mitochondrial respiration rate, calculated as $R_m = 0.5 \times R_d$ (von Caemmerer, 2000), $P_m$ is CO₂ partial pressure in the mesophyll cells (assumed to equal measured $P_i$), $\gamma^*$ is half of the reciprocal of Rubisco specificity (0.000193) (von Caemmerer et al., 1994), $O_m$ is the O₂ partial pressure in the mesophyll cells (19.53 KPa), $\alpha$ is the fraction of PSII activity in the bundle-sheath cells ($0 \leq \alpha \leq 1$; $\alpha = 0$ for Z. mays and M. giganteus), $x$ is the portion of ATP required for the C₄ cycle, assumed to equal 0.4 under RGB light and variable under the other light qualities (von Caemmerer, 2000) and $g_{bs}$ (µmol m⁻² s⁻¹ Pa⁻¹) is bundle-sheath conductance to CO₂ determined as the value that best matched predicted and observed photosynthetic discriminations using the data from the first hour RGB light measurements (Kromdijk
et al., 2010; Ubierna et al., 2011; Sun et al., 2012; Ubierna et al., 2013). For other light qualities $g_{bs}$ was assumed constant and equal to the value under RGB light and the photosynthetic discrimination model was solved for $x$.

Substituting values for $a$, $b$ and $c$ in Eqn S1 and solving for $J_t$ results in:

$$IIIJ_t^2 + IIJ_t + I = 0 \quad (\text{Eqn S5})$$

and thus

$$J_t = \frac{-II + \sqrt{II^2 - 4III 	imes I}}{2III} \quad (\text{Eqn S6})$$

where I, II and III are defined as:

$$I = \left(1 + \frac{R_d}{A}\right) \times \left(\frac{R_m - g_{bs}P_m - \frac{7g_{bs}\gamma^*O_m}{3}}{3} \right) + (R_d + A) \times \left(1 - \frac{7\alpha\gamma^*}{3 \times 0.047}\right) \quad (\text{Eqn S7})$$

$$II = \frac{1 - x}{3} \left[\frac{g_{bs}}{A} \left(\frac{P_m - R_m - \gamma^*O_m}{g_{bs}}\right) - 1 - \frac{\alpha\gamma^*}{0.047}\right] - \frac{x}{2} \left[1 + \frac{R_d}{A}\right] \quad (\text{Eqn S8})$$

$$III = \frac{x - x^2}{6A} \quad (\text{Eqn S9})$$

Once $J_t$ was derived, C$_4$ photosynthetic parameters were calculated using von Caemmerer (2000) light-limited C$_4$ photosynthesis model. In particular, $P_{bs}$, the CO$_2$ partial pressure inside the bundle-sheath cells was calculated as:

$$P_{bs} = \frac{(\gamma^*O_s) \left[\frac{7}{3} \left(A + R_d\right) + \frac{(1-x)J_t}{3}\right]}{(1-x)J_t} \frac{3}{3} - (A + R_d) \quad (\text{Eqn S10}) \quad (\text{Eqn 4.39, von Caemmerer, 2000})$$

where $O_s$ is O$_2$ partial pressure in the bundle-sheath cells, calculated as:

$$O_s = \frac{\alpha A}{0.047g_{bs}} + O_m \quad (\text{Eqn S11}) \quad (\text{Eqn 4.16, von Caemmerer, 2000})$$
References:


